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Model Predictive Control Allocation for Overactuated Systems - Stability and Performance

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Abstract— **Overactuated systems often arise in automotive, aerospace, and robotics applications, where for reasons of redundancy or performance constraints, it is beneficial to equip a system with more control inputs than outputs. This necessitates control allocation methods that distribute control effort amongst many actuators to achieve a desired effect. Until recently, most methods have treated the control allocation as static in the sense that different dynamic authorities of the actuators were not taken into account. Recent advances have used model predictive control allocation (MPCA) to consider the dynamic authorities of the actuators over a receding horizon. In this paper, we consider the dynamic control allocation problem for overactuated systems where each actuator has different dynamic control authority and hard saturation limits. A modular control design approach is proposed, where the controller consists of an outer loop controller that synthesizes a desired virtual control input signal and an inner loop controller that uses MPCA to achieve the desired virtual control signal. We derive sufficient stability conditions for the composite feedback system and show how these conditions may be realized by imposing an additional constraint on the MPCA design. An automotive example is provided to illustrate the effectiveness of the proposed algorithm.**

I. INTRODUCTION

Overactuated systems frequently arise in robotics, aerospace, marine, and automotive applications, for various reasons. In some instances, a fast actuator is used for good transient response, but a slow actuator is used for steady-state operation in order to improve efficiency. Idle speed control of an internal combustion engine is an example, where both spark advance and bypass valve are used to control engine speed. In some other cases, certain actuators lose authority under particular operating conditions, necessitating a complementary actuator to extend the range of operation (robotic manipulators are an example of this). There are also cases where the overactuation is simply a matter of redundancy.

For an overactuated system, one can often identify a signal that characterizes the overall effect (such as a force or moment) of many actuators, which acts as a "virtual control." For a MISO (multi-input, single output) system, the following representation can be used to decompose the system into two subsystems after introducing the virtual control input:

$$\begin{aligned} x_1(k+1) &= f_1(x_1(k), v(k)) \\ x_2(k+1) &= f_2(x_1'(k), x_2(k), u(k)) \\ v &= g(x_2, u) \\ y &= h(x_1), \end{aligned} \tag{1}$$

where $u \in \mathbb{R}^q$, $y \in \mathbb{R}$, and $v \in \mathbb{R}$ represent the control inputs, the performance output, and the *virtual control input*, respectively [1]. Furthermore, each control input is subject to hard saturation limits as:

$$u \in U = \{u : u_{min,i} \leq u_i \leq u_{max,i}, \forall i\}. \tag{2}$$

It should be noted that $x_1 \in \mathbb{R}^{n_1}$ contains the plant states, which are driven by the virtual control input, v, whereas $x_2 \in \mathbb{R}^{n_2}$ contains the actuator states, which are driven by the real control inputs, u. $x_1' \in \mathbb{R}^{n_1'}$, with $n_1' \leq n_1$, represents a subset of plant states that affect the actuator dynamics. The real control inputs, u, affect the plant states only through the virtual control input, v.

The introduction of the virtual control input to decompose the system into two parts naturally leads to a modular control strategy, as depicted in Fig. 1, where the objective is to have y track a setpoint, r. Here, an outer loop controller determines a desired virtual control input, v_{des}, and an inner loop controller determines the real control inputs, u, that will be applied to the actuators in order to achieve v_{des} as closely as possible. This modular approach is very commonly employed in practice, allowing the overall control design to be divided amongst different areas of expertise. When the modular approach is applied to an overactuated system, the inner loop control design problem is commonly referred to as *control allocation*.

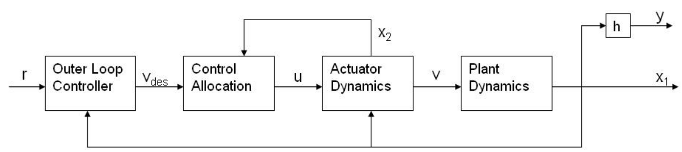

Fig. 1. Block Diagram of the Overall System Under a Modular Control Strategy

Until recently, it was assumed, as in [1] - [5], that each actuator had an immediate effect on the virtual control input and possessed hard saturation limits. Because of these

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[1]We consider MISO case in this paper, with a single virtual control input. However, the methodology we develop can be directly extended to general MIMO systems, where y and v are vectors.

assumptions, only saturation could prevent the desired virtual control input from being satisfied at all times. The authors of [6] postprocess the output of a static control allocation in an attempt to compensate for actuator dynamics but do not optimize control inputs to take full advantage of different dynamic authorities. [7] - [9] are some of the first to consider the case when actuators do not possess the same dynamic authorities, using attitude control on a re-entry vehicle as a case study. In [7] - [9], the authors use an inner-loop/outer-loop control strategy where the outer loop controller determines a desired virtual control input, and the inner loop uses model predictive control allocation (MPCA) to achieve this virtual control input trajectory as tightly as possible over a receding horizon. Because this strategy distinguishes fast actuators from slow actuators, it yields considerable performance advantages over previous strategies.

While [7] - [9] represent significant advances in control allocation, they do not provide a generalized stability guarantee for the overall system given by Fig. 1 ([8] and [9] do provide a stability guarantee for the inner loop alone). This paper extends the results in MPCA by proposing an augmentation to the MPCA that guarantees input-to-state stability (ISS) of the overall system while preserving performance.

Consider an inner loop controller that is synthesized by optimizing the following cost function over a receding horizon:

$$J(x_2(k),u) = \sum_{i=k}^{k+N-1} ((v_{des}(i)-v(i))^2 + P(u(i),x_2(i))), \tag{3}$$

where N is the length of the prediction horizon. The first term in the cost function penalizes deviation between actual and desired virtual control inputs, whereas the second is an optional term which can be used to shape the response of the closed-loop system. In this paper, we will develop an additional constraint that can be enforced by the MPCA in order to guarantee stability. [2]

The paper is organized as follows. In Section II, we derive the stability requirements for modular (control allocation-based) control of overactuated systems and use these results to provide the stability augmentation that will guarantee stability of the overall system with MPCA. In Section III, we provide a simple example that illustrates the workings of the method shown in Section II. In Section IV, we apply the method to a thermal management system, illustrating the effectiveness and flexibility of the proposed approach.

II. CLOSED-LOOP STABILITY REQUIREMENTS WITH MODULAR CONTROL

For general nonlinear systems represented by (1), many different notions of stability can be applied, which will lead to different implications. In this paper, we consider "l_2 to l_2" input-to-state stability (ISS [3]), as defined in [12], and applied to the discrete-time system considered here, in terms of the specific input (r) and output (y) of the system depicted in Fig. 1:

Definition 2.1: (Input-to-State Stability) The system given in Fig. 1, whose open-loop dynamics are given by (1), is ISS if $\exists$ a class K function β and a class K_∞ function γ_{iss} [4] such that:

$$\sum_{i=0}^{n} \|x(i)\|_2^2 \leq \beta(\|x(0)\|_2^2) + \sum_{i=0}^{n} \gamma_{iss}(|r(i)|^2), \forall n \geq 0, \tag{4}$$

where $x = (\; x_1^T \quad x_2^T)^T$.

□

Because we will be studying the stability of the system under the modular, inner-loop/outer-loop strategy proposed in Section I, we cast the system (1), with both the inner and outer loop controllers in place, as two interconnected systems, depicted in Fig. 2 and described by:

$$\Sigma_1 := \begin{cases} x_1(k+1) = f_1(x_1(k), v_{des}^f(k), \tilde{v}(k)) \\ v_{des}(k) = c_1(x_1(k), r(k)) \\ x_1'(k) = d(x_1(k)) \end{cases} \tag{5}$$

$$\Sigma_2 := \begin{cases} x_2(k+1) = f_2(x_1'(k), x_2(k), u(k)) \\ u(k) = c_2(x_1'(k), x_2(k), v_{des}(k)) \\ v(k) = g(x_2(k), u(k)) \\ \tilde{v}(k) = v(k) - v_{des}^f(k) \end{cases} \tag{6}$$

$$F := \begin{cases} x_f(k+1) = f_3(x_f(k), v_{des}(k)) \\ v_{des}^f(k) = g_f(x_f(k)) \end{cases} \tag{7}$$

where Σ_1 and Σ_2 denote the outer loop and inner loop systems with their corresponding controllers, $c_1(x_1(k), r(k))$ and $c_2(x_1'(k), x_2(k), v_{des}(k))$, respectively. For this paper, c_2 will be synthesized using MPCA. Given a well-defined relative degree, ρ, from u to v, F is a filter with relative degree ρ (in the simplest case, a ρ step time delay). The presence of F will lead to a system representation which, for future stability analysis, will allow the small gain condition (a sufficient condition for stability) to be satisfied with a tracking/disturbance-rejecting outer loop and causal inner loop controller.

Given this system representation, and from the results derived in [11] and [12] for general interconnected systems (which fit the framework depicted in Fig. 2), the following Proposition yields sufficient conditions for ISS of the composite system:

Proposition 2.1: (Closed-Loop ISS Conditions with the Modular Inner-Loop/Outer-Loop Controller) Suppose that Σ_1 is ISS from inputs r and $\tilde{v}$ to states x_1 and Σ_2 is ISS from inputs x_1' and v_{des} to states x_2. Namely, there exist

[2]The cost, $J(x_2(k),u)$, and the optimal control input, u^*, will, in general, depend on $v_{des}(i)$ and $x_1'(i)$, $i = k \dots k+N-1$. These quantities can either be assumed constant over the entire MPCA prediction horizon or can be estimated for $i > k$, using the plant model.

[3]For the remainder of the paper, ISS will always refer to "l_2 to l_2" input-to-state stability.

[4]A continuous function, $f(x) : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ belongs to class K_∞ iff $f(0) = 0$ and $f(x)$ is strictly increasing in x and unbounded from above. A continuous function, $g(x) : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ belongs to class K iff $g(0) = 0$ and $g(x)$ is strictly increasing (but not necessarily unbounded) in x.

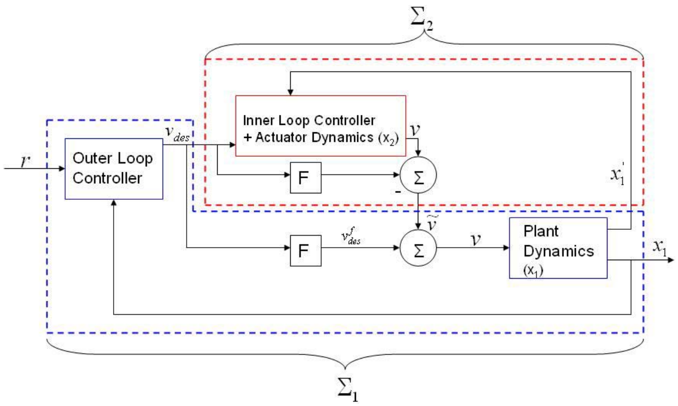

Fig. 2. Full Diagram of the System (1) Recast for Stability Analysis

class *K* functions $\beta_{1,2}$ and class K_∞ functions γ_{iss1}^r, γ_{iss1}^y, and γ_{iss2}^y that satisfy:

$$\sum_{i=0}^{n} \|x_1(i)\|_2^2 \le \beta_1(\|x_1(0)\|_2^2) \tag{8}$$
$$+\sum_{i=0}^{n} \gamma_{iss1}^r(|r(i)|^2) + \sum_{i=0}^{n} \gamma_{iss1}^y(|\tilde{v}(i)|^2), \forall n \ge 0,$$
$$\sum_{i=0}^{n} \|x_2(i)\|_2^2 \le \beta_2(\|x_2(0)\|_2^2) \tag{9}$$
$$+\sum_{i=0}^{n} \gamma_{iss2}^y(\|\begin{pmatrix} v_{des}(i) & x_1^{'T}(i) \end{pmatrix}\|_2^2), \forall n \ge 0.$$

Also, assume that there exist class *K* functions $\beta_{1,2}^y$ and K_∞ functions γ_1^r, γ_1^y, and γ_2^y that satisfy the following small gain condition:

$$\gamma_1^y \circ \gamma_2^y(s) < s, \forall s > 0, \tag{10}$$

where $\beta_{1,2}^y$, γ_1^r, γ_1^y, and γ_2^y satisfy:

$$\sum_{i=0}^{n} \|\begin{pmatrix} v_{des}(i) & x_1^{'T}(i) \end{pmatrix}\|_2^2 \le \beta_1^y(\|x_1(0)\|_2^2) \tag{11}$$
$$+\sum_{i=0}^{n} \gamma_1^r(|r(i)|^2) + \sum_{i=0}^{n} \gamma_1^y(|\tilde{v}(i)|^2), \forall n \ge 0,$$
$$\sum_{i=0}^{n} |\tilde{v}(i)|^2 \le \beta_2^y(\|x_2(0)\|_2^2) \tag{12}$$
$$+\sum_{i=0}^{n} \gamma_2^y(\|\begin{pmatrix} v_{des}(i) & x_1^{'T}(i) \end{pmatrix}\|_2^2), \forall n \ge 0.$$

Then the composite system described by (1) is ISS from r to x.

□

In Proposition 2.1, Equations (8) and (9) relate the inputs to the states of the individual subsystems, whereas Equations (11) and (12) relate the inputs to the outputs and are needed in order to verify the small gain condition (10). The proof follows from that given for generic interconnected systems in [11]. It is worth noting that the small gain property (10) is merely a sufficient condition for stability and therefore may represent a conservative bound.

While Proposition 2.1 gives the conditions for ISS of the system described in (1), these conditions cannot serve as a method for verifying the ISS property. The following Proposition [12], presented for a generic discrete-time system with state variable x and input u, provides a method of verifying the ISS condition and deriving an input-to-state function, γ_{iss}:

Proposition 2.2: (Verifying ISS for General Systems) Given a generic discrete time system, $x(k+1) = f(x(k), u(k))$, suppose that there exists a Lyapunov function, $V(x)$, and class K_∞ functions α_1 and α_2 that satisfy:

$$\alpha_1(\|x\|_2^2) \le V(x) \le \alpha_2(\|x\|_2^2). \tag{13}$$

Furthermore, assume that there exists a class K_∞ function χ such that:

$$\|u\|_2^2 \le \chi(\|x\|_2^2) \Longrightarrow V(x(k+1)) - V(x(k)) < 0. \tag{14}$$

Then the system is ISS with input-to-state class K_∞ function defined by:

$$\gamma_{iss} = \alpha_1^{-1} \circ \alpha_2 \circ \chi^{-1}. \tag{15}$$

□

The method detailed in Proposition 2.2 may be applied to each individual system, Σ_1 and Σ_2, described in (5) and (6) and depicted in Fig. 2 by substituting the appropriate states (x_1 or x_2) and the appropriate inputs ($\tilde{v}$ or $\begin{pmatrix} v_{des} & x_1^{'T} \end{pmatrix}$), respectively, for the generic states x and inputs u used in the proposition.

Remark 2.1: γ_{iss}, the *input-to-state* function may be used in conjunction with the state-to-output mapping for the system in order to determine γ, the *input-output* function.

Given this method for verifying ISS, we may now proceed to define an algorithm that may be used in order to augment the MPCA to guarantee ISS for the system at hand. The following *ISS-Constrained Model Predictive Control Allocation* algorithm is proposed in order to achieve ISS for the composite system:

1) Design an outer loop controller, $v_{des}(k) = c_1(x_1(k), r(k))$, that renders the outer loop system ISS.
2) Design a stabilizing inner loop controller, $u_r(k) = c_{ref}(x_1'(k), x_2(k), v_{des}(k))$ and associated control Lyapunov function, $V(x_2)$, that satisfies the conditions set forth by Proposition 2.2 and thereby renders the inner loop system ISS. Furthermore, it must yield an input-output class K_∞ function, $\gamma_{2,ref}^y$, such that $\gamma_1^y \circ \gamma_{2,ref}^y(s) < s$. This inner loop controller will be known as the *reference controller*.
3) Let $u_m(i)$ and $u_r(i)$ be the control inputs that are calculated by the MPCA design and reference controller, respectively. Place the following constraints on the first step of each MPCA control sequence:
   a) $V(x_2(k+1))|_{u_m(k)} \le V(x_2(k+1))|_{u_r(k)}$,
   b) $|\tilde{v}(k+\rho)|_{u_m(k)} \le |\tilde{v}(k+\rho)|_{u_r(k)}$.

Proposition 2.3: (Stability Augmentation for Model Predictive Control Allocation) The closed-loop system (5),(6), with inner and outer loop controllers designed following the *ISS-Constrained Model Predictive Control Allocation* algorithm above, is ISS.

□

The formal proof is omitted for brevity. The first constraint (3a) ensures ISS of the inner loop alone, and the second constraint (3b) ensures that the small gain condition is satisfied when u_m is used instead of u_r. The reference controller needs only to stabilize the system and may, in fact, result in poor performance. On the other hand, the controller rendered by MPCA can assure good performance by minimizing a suitable cost function.

III. AN ILLUSTRATIVE EXAMPLE

Consider the following linear system:

$$Y(s) = \frac{1}{s^2+2s+1}V(s), \qquad (16)$$
$$V(s) = \frac{1}{.2s+1}U_1(s) + \frac{1}{.2s^2+1.2s+1}U_2(s),$$

where the notation is consistent with (1), and the objective of the control system is to have y track a reference signal r. When (16) is discretized with a sampling time of 0.05s and a zero-order hold, it yields the discrete-time system:

$$Y(z) = \frac{.001209z+.001169}{z^2-1.902z+.9048}V(z), \qquad (17)$$
$$V(z) = \frac{.2212}{z-.7788}U_1(z) + \frac{.005663z+.005125}{z^2-1.73z+.7408}U_2(z).$$

A. Control Design

For the outer loop, we use the controller:

$$V_{des}(z) = \frac{2z-1.95}{z-1}(R(z)-Y(z)). \qquad (18)$$

For the inner loop *reference controller*, we choose $u_1 = v_{des}, u_2 = 0$. Note that MPCA may still utilize both control inputs even though the reference controller only utilizes u_1. Taking $F(z) = z^{-1}$, overall system stability can be confirmed via the small gain property, with $\gamma_1^y(s) = 1.07s$ and $\gamma_2^y(s) = 0.88s$. We use the control Lyapunov function, $V(x_2) = x_2^T P x_2$, to verify inner loop stability, where P is the solution to the discrete-time Lyapunov equation, $A^T PA - P = -I$, and A is the system matrix of the *closed* inner loop system (Σ_2), namely:

$$\begin{aligned} x_2(k+1) &= Ax_2(k) + Bv_{des}(k), \\ v &= Cx_2. \end{aligned} \qquad (19)$$

$V(x_2)$ is also used to enforce constraint (3a) in the MPCA algorithm described in Section II.

The MPCA cost function for this example is taken as:

$$J(x_2(k), u) = \sum_{i=k}^{k+N-1} (v_{des}(i) - v(i))^2. \qquad (20)$$

The time step used for the MPCA simulation is the same as the discretization step (0.05s), and the horizon time is 0.5s (10 steps).

B. Simulation Results

Fig. 3 compares the output responses of the closed-loop system under MPCA with and without the stability augmentation, as well as the performance under the reference controller. MPCA with the stability augmentation outperforms both the reference controller and the MPCA without the augmentation. In fact, Fig. 3 does not indicate that MPCA without the stability augmentation leads to a stable closed-loop system.

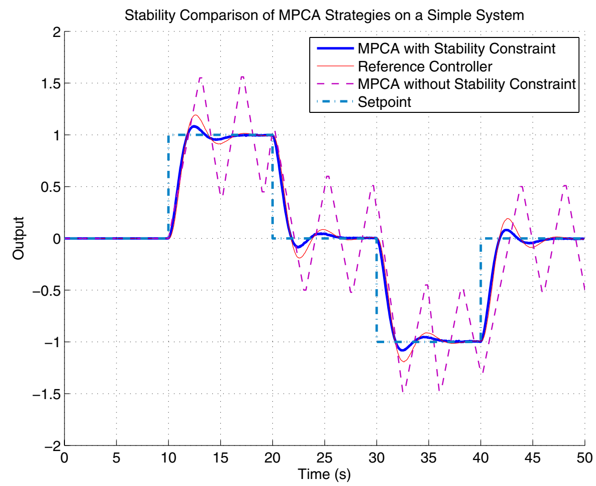

Fig. 3. Comparison of Performance Under Standard and Improved MPCA Strategies

Fig. 4 shows the control inputs, demonstrating that without the stability augmentation, the control inputs oscillate between the upper and lower limits used by the optimization algorithm in its search for u. Because of the actuator dynamics, the control inputs do not affect the virtual control input immediately, and indeed, do not have a large effect on the virtual control input for much of the receding horizon. Therefore, the actuators overcompensate in order to minimize the MPCA cost function over a relatively short horizon, which does not lead to a stable closed-loop system.

IV. APPLICATION TO ENGINE THERMAL MANAGEMENT

The proposed modular control design approach is now applied to an engine thermal management system, depicted in Fig. 5. The system is used to facilitate engine mapping and calibration on a dynamometer by providing tight regulation of coolant and oil temperatures at the outlet of an automotive engine. While the physical system consists of two parallel loops (coolant and oil), [10] has shown that the interaction between the two is minimal, and it is in fact possible to decouple the two in control analysis and design. In this study, we will consider only the coolant loop, as depicted in Fig. 5. Each loop includes two actuators (a mixing valve and a heater), which are used to control the temperature of the fluid at the engine outlet. The application of dynamic control allocation via MPCA is especially attractive for this

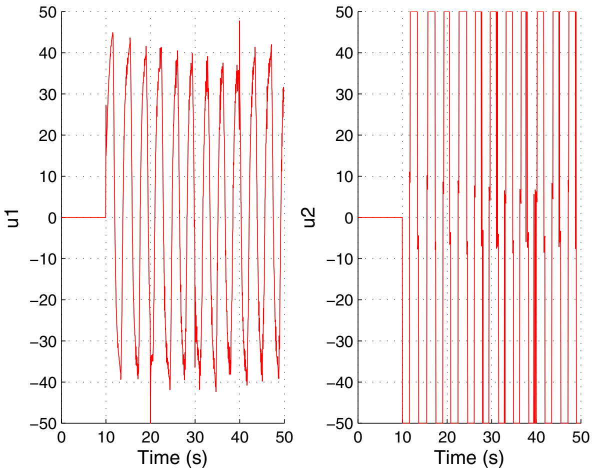

Fig. 4. Control Inputs Under Standard MPCA

system since, as seen from the analysis presented in [10], the two actuators possess very different dynamic authorities (the mixing valve serves as the "fast" actuator).

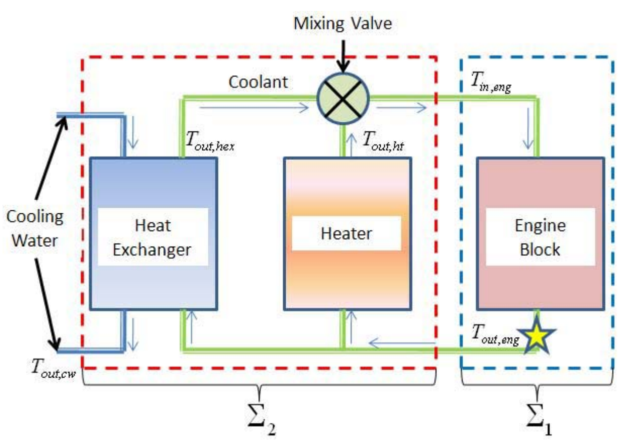

Fig. 5. Thermal Management System Diagram

A. System Description and Formulation for Stability Analysis

The dynamic model for this system is detailed in [10]. From the mixing valve command, u_{mv}, we are able to control the *bypass ratio*, θ, where $\theta = W_{ht}/W$ is defined as the ratio of the flow rate through the heater to the total flow rate (W). Similarly, from the heater command, u_{ht}, we may uniquely determine the power supplied by the heater, $\dot{Q}_{ht}$.

In the physical system, the heater, heat exchanger, and mixing valve all reside in a unit that is separated from the engine. This unit naturally forms the actuator subsystem, Σ_2. The engine block (including the fluid that passes through it) comprises the plant subsystem, Σ_1. Therefore, the thermal management system of Fig. 5 can be cast in the form of (1), with three plant states (temperature of fluid exiting the engine block ($T_{out,eng}$) and two engine block temperatures (T_{eng1} and T_{eng2}) to represent different temperatures in different regions of the engine) and four actuator states (temperature of fluid exiting the heater ($T_{out,ht}$), heater coil temperature (T_{ht}), temperature of the fluid exiting the heat exchanger ($T_{out,hex}$), and temperature of the cooling water exiting the heat exchanger ($T_{out,cw}$)). The virtual control input is the engine inlet temperature, $T_{in,eng}$.

For stability analysis, we will consider the deviations, denoted by δ, of the temperatures about nominal temperatures, T^*. The plant states are given by $x_1 = \begin{pmatrix} \delta T_{out,eng} & \delta T_{eng1} & \delta T_{eng2} \end{pmatrix}^T$, and the actuator states are given by $x_2 = \begin{pmatrix} \delta T_{out,ht} & \delta T_{ht} & \delta T_{out,hex} & \delta T_{out,cw} \end{pmatrix}^T$. Finally, $x_1' = \delta T_{out,eng}$. The virtual control input, control inputs, and performance variable are given by $v = \delta T_{in,eng}$, $u_1 = \delta\theta$, $u_2 = \delta \dot{Q}_{ht}$, and $y = \delta T_{out,eng}$, respectively.

B. Control Design and Stability Analysis

The outer loop system, which is linear for a given flow rate, may be controlled effectively (and rendered ISS) with a simple PI controller, $v_{des}(k) = r(k) - y(k) + \sum_{i=1}^{k} 0.1T(r(k) - y(k))$ (temperature is measured in degrees Celsius, time is in seconds, and T is the sampling time). For the inner loop reference controller, we choose the control law $u_{1,ref} = k_o v_{des} + k_i x_2$, $u_{2,ref} = u_{max}$, where $k_o = 0.02, k_i = \begin{pmatrix} 0.02 & 0 & 0 & 0 \end{pmatrix}$. This choice of a controller can be shown, through a control Lyapunov function, $V(x_2) = ax_{21}^2 + bx_{22}^2 + cx_{23}^2 + dx_{24}^2$ to guarantee ISS for the inner loop.

Having verified ISS for both the inner and outer loops, we must now verify that the small gain condition holds. Because the relative degree from u to v is equal to 0, it may be possible to satisfy the small gain theorem with $F = 1$ (i.e., no filtering of v_{des} to compute $\tilde{v}$). This leads to $\gamma_1^y(s) \approx s$ for the outer loop (the smallest possible input-output gain with an integrator). In order to derive an input-output class K_∞ function for the inner loop, $\gamma_{iss2}^y(s)$ must be related to $\gamma_2^y(s)$ through the input-output relation,

$$T_{in,eng} = \theta T_{out,ht} + (1-\theta)T_{out,hex}. \tag{21}$$

This relation is used to derive an expression for $\tilde{v}$ in translated coordinates:

$$\begin{aligned} \tilde{v} &= v_{des} - v \\ &= v_{des} - u_1(T_{out,ht}^* - T_{out,hex}^*) - \theta^* x_{21} \\ &\quad -(1-\theta^*)x_{23} - u_1(x_{21} - x_{23}), \end{aligned} \tag{22}$$

where the final expression is obtained by substituting the inner loop reference control law for v_{des} and using (21) to express v in terms of x_2 and u_{mv}. In order to use this relation to verify the small gain property, we must substitute the inner loop reference control law for u, incorporating saturation limits of the actuators, i.e.,

$$u_{1,ref,sat} = \begin{cases} u_{1,min}, & u_{1,ref} < u_{1,min} \\ u_{1,max}, & u_{1,ref} > u_{1,max} \\ u_{1,ref}, & otherwise \end{cases} \tag{23}$$

Because of these saturation limits, we may not allow v_{des} to become arbitrarily large and guarantee an input-output gain less than 1. However, if we constrain the output of the outer loop, v_{des}, so that it does not cause saturation of the mixing valve through the feedforward term ($k_o v_{des}$) of the reference controller, namely requiring that $u_{min} \leq k_o v_{des} \leq u_{max}$ (a saturation constraint on v_{des}), then the inner loop control law for u, incorporating saturation limits, may be written as $u_{1,ref} = 0.02v_{des} + sat(0.02x_{21})$, $u_{2,ref} = 0$, such that we may rewrite (22) as:

$$\begin{aligned} \tilde{v} \;=\; & v_{des}(1 - 0.02(T^*_{out,ht} - T^*_{out,hex})) \\ & -sat(0.02x_{21})(T^*_{out,ht} - T^*_{out,hex}) - \\ & (\theta^* + u_{1,ref,sat})x_{21} + (\theta^* + u_{1,ref,sat} - 1)x_{23}. \end{aligned} \tag{24}$$

Recognizing saturation constraints on the valve input, u_1, we may now derive an inner loop input-output class K_∞ function, $\gamma_2^y(s) = 0.6s$. Composition of $\gamma_1^y(s)$ with $\gamma_2^y(s)$ satisfies the small gain property. For the trajectories we consider in simulation, v_{des} stays within its limits, so the constraint on v_{des} is not active. In other situations, methods exist for estimating the input-output gain of the system with limits on v_{des} [13].

C. Simulation Results

Results of applying the constrained MPCA, with a cost function (20), are shown in Fig. 6. Imposing a constraint by the reference controller guarantees that the overall system is stable, and the use of MPCA ensures good performance through minimization of a cost function. Moreover, *the stability guarantee is independent of the choice of cost function*. In this particular example, we are able to attain some additional performance advantage by penalizing deviation from preferred actuator settings, u_p, in addition to the basic penalty on deviation between v_{des} and v. This penalty ensures that, once transients have settled, the fast actuator (the mixing valve) possesses sufficient reserve capability to track setpoints. This is evidenced in simulation results, where the temperature increase poses a challenge due to the low speed and load at which the simulation was run.

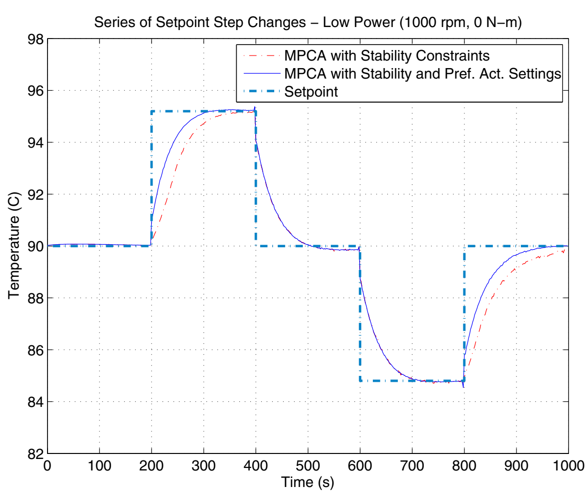

Fig. 6. MPCA for the Thermal Management System

V. CONCLUSIONS AND FUTURE WORK

In this work, we have derived sufficient conditions for the input-to-state stability of systems where control allocation is used to achieve a desired virtual control input. We have used these conditions to place a constraint on the actions taken by model predictive control allocation (MPCA), such that the MPCA may be employed in order to optimize system performance through proper choice of cost function, while guaranteeing stability of the overall system. This has been demonstrated through simulation on a simple example as well as a physical engine thermal management system. Future work should aim to address computational complexity of the MPCA optimization, introduce systematic designs for constructing stabilizing inner loop reference controllers in the presence of constraints, and reduce conservatism of MPCA stability constraints.

REFERENCES

[1] M. Naghshineh and M. Keshumiri, "Actuator Saturation Avoidance in Overactuated Systems," *Proceedings of the 2004 IEEE/RSJ International Conference on Intelligent Robots*.

[2] K. Ohishi, H. Nozawa, T. Miyazaki, "Redundant Manipulator Control with Autonomous Consideration Algorithm of Torque Saturation," ISIE 1999.

[3] O. Harkegard, "Resolving Actuator Redundancy - Control Allocation vs. Linear Quadratic Control," ECC 2003.

[4] M. Bodson, "Evaluation of Optimization Methods for Control Allocation," *Journal of Guidance, Control, and Dynamics*, Vol. 25, No. 4, July-August 2002.

[5] W. Durham, "Attainable Moments for the Contstrained Control Allocation Problem," *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 6, 1994.

[6] M. Oppenheimer, D. Doman, "Methods for Compensating for Control Allocator and Actuator Interactions," *Journal of Guidance, Control, and Dynamics*, Vol. 27, No. 5, 2004.

[7] Y. Luo, A. Serrani, S. Yurkovich, D. Doman, M. Oppenheimer, "Model Predictive Dynamic Control Allocation with Actuator Dynamics," *Proceedings of the American Control Conference*, 2004.

[8] Y. Luo, A. Serrani, S. Yurkovich, D. Doman, M. Oppenheimer, "Dynamic Control Allocation with Asymptotic Tracking of Time-Varying Control Input Commands," *Proceedings of the American Control Conference*, 2005.

[9] Y. Luo, A. Serrani, S. Yurkovich, M. Oppenheimer, D. Doman, "Model Predictive Dynamic Control Allocation Scheme for Reentry Vehicles," *Journal of Guidance, Control, and Dynamics*, Vol. 30, No. 1, 2007.

[10] C. Vermillion, J. Sun, K. Butts, A. Hall, "Modeling and Analysis of a Thermal Management System for Engine Calibration," *Proceedings of the IEEE Conference on Control Applications*, 2006.

[11] Z.-P. Jiang, A. R. Teel, L. Praly, "Small-Gain Theorem for ISS Systems and Applications," *Mathematics of Control, Signals, and Systems*, pp. 95-120, 1994.

[12] E. Sontag, "Comments on Integral Variants of ISS," *Systems and Control Letters*, Vol. 34, pp. 93-100, 1998.

[13] T. Hu, A. Teel, L. Zaccarian, "Nonlinear L_2 Gain and Regional Analysis for Linear Systems with Anti-windup Compensation," *Proceedings of the American Control Conference*, 2005.