### Systems biology

# **NET-SYNTHESIS:** a software for synthesis, inference and simplification of signal transduction networks

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ABSTRACT

**Summary:** We present a software for combined synthesis, inference and simplification of signal transduction networks. The main idea of our method lies in representing observed indirect causal relationships as network paths and using techniques from combinatorial optimization to find the sparsest graph consistent with all experimental observations. We illustrate the biological usability of our software by applying it to a previously published signal transduction network and by using it to synthesize and simplify a novel network corresponding to activation-induced cell death in large granular lymphocyte leukemia.

Availability: NET-SYNTHESIS is freely downloadable from http:// www.cs.uic.edu/~dasgupta/network-synthesis/

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**Supplementary information:** Supplementary data are available at *Bioinformatics* online.

### **1 INTRODUCTION**

Identification of every reaction and regulatory interaction participating even in a relatively simple function of a singlecelled organism requires a concerted and decades-long effort. Consequently, the state of the art understanding of many signaling processes is limited to the knowledge of key mediators and of their positive or negative effects on the whole process. For example, evidence of differential responses to a stimulus in wild-type organisms versus a mutant organism implicates the product of the mutated gene in the signal transduction process. The resulting causal inference relates three components (the signal, the mutated gene and the response) and only in a minority of cases corresponds to a single reaction (namely, when the stimulus is the reactant of the reaction, the mutated gene encodes the enzyme catalyzing the reaction and the studied output is the product of the reaction). We previously introduced (Albert et al., 2007) a method of synthesizing interactions and causal inferences into a parsimonious network

by incorporating positive (activating) or negative (inhibitory) causal relationships as signed network paths with known starting and end vertices (nodes) and putative intermediary pseudonodes. Here, we describe an automated version of the method available for use by the community.

### 2 SOFTWARE OVERVIEW

Our software uses as input a text file whose lines represent causal relationships such as  $A \rightarrow B'$  (representing activation), 'A  $\dashv$  B' (representing inhibition), or 'A  $\rightarrow$  (B  $\dashv$  C)' (indicating a double causal inference). Relationships that correspond to direct interactions are specified by the label 'Y', e.g. ' $A \rightarrow B$  Y'. In addition, the relationship between the enzyme (E) and product (P) of a chemical reaction (i.e.  $(E \rightarrow P)$ ) is labeled both 'Y' and 'E' (for enzymatic edge). The entire network synthesis procedure is given in the Supplementary Material; here we briefly describe some key steps. Double causal relationships of the form A x (B y C) with  $x, y \in \{\rightarrow, \dashv\}$  are represented by adding a new 'pseudo-vertex' P and three new edges, A x P, B a P and P b C, where a and b are determined by y. Two graphtheoretic procedures, the pseudo-vertex collapse (PVC) and binary transitive reduction (BTR), are used as key steps in the algorithm. Intuitively, the PVC problem is useful for reducing the pseudo-vertex set to the minimal set that maintains the graph consistent with all indirect experimental observations and the BTR problem is useful for determining a sparsest graph consistent with all experimental observations. Although the initial motivation for introducing pseudonodes is to represent the intersection of the two paths corresponding to three-node inferences, PVC can be used in the broader context of network simplification. In many large-scale regulatory networks only a subset of the nodes are of inherent interest, e.g. because they are differentially expressed in different exogenous conditions, and the rest serve as background or mediators. Our software enables users to designate vertices of less interest or confidence as pseudo-vertices and then collapse them, thereby making the network among high-interest/confidence nodes

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easier to interpret. To allow gradual simplification, we also provide the choice to collapse degree two pseudonodes only or only collapse one pair of equivalent pseudo-vertices. A detailed manual of the software is available from the software's website. The software should run on any machine with MS Windows (Win32). The source files for a non-graphic version of the program for LINUX/UNIX systems can be obtained by sending an email to the authors.

### 2.1 Data sources

Large-scale repositories such as Many Microbe Microarrays (http://m3d.bu.edu/cgi-bin/web/array/index.pl?read =aboutM3D), NASCArrays (http://affymetrix.arabidop sis.info/narrays/experimentbrowse.pl) and Gene Expression Omnibus (http://www.ncbi.nlm.nih.gov/geo/) contain expression information for thousands of genes under tens to hundreds of experimental conditions. Network inference algorithms applied to gene expression data based on, e.g. mutual information, regression or Bayesian analysis lead to indirect causal relationships among genes. NET-SYNTHESIS can be used to filter redundant inferred relationships by binary transitive reduction. In addition, information about differentially expressed genes responding to a combination of two experimental perturbations, e.g. the presence of a signal in normal versus mutant organisms, can be expressed as double causal inferences. NET-SYNTHESIS can be used to interpret these inferences by pseudo-vertex collapse. Signal transduction pathway repositories such as TRANSPATH (http://www.generegulation.com/pub/databases.html#transpath) and protein interaction databases such as the Search Tool for the Retrieval of Interacting Proteins (http://string.embl.de/) contain up to thousands of interactions, a large number of which are not supported by direct physical evidence. NET-SYNTHESIS can be used to filter redundant information while keeping all direct interactions.

### **3 RESULTS AND DISCUSSIONS**

# 3.1 Synthesizing a network for T-cell survival and death in large granular lymphocyte leukemia

T-cell large granular lymphocyte leukemia (T-LGL) represents a spectrum of lympho-proliferative diseases in which cytotoxic T lymphocyte activation and elimination are uncoupled (Loughran, 1993). To date, 33 proteins and small molecules related to cytotoxic T lymphocyte activation and activationinduced cell death have been shown to be deregulated in T-LGL and it is known that pro-survival signaling pathways are upregulated and that T-LGL cells are insensitive to Fas-induced apoptosis (Epling-Burnette *et al.*, 2004). However, the interaction/regulatory network among these components remains largely unknown.

We synthesized a cell-survival/cell-death regulation-related signaling network from the TRANSPATH 6.0 database with additional information manually curated from literature search. The 359 vertices of this network represent proteins/ protein families and mRNAs participating in pro-survival and Fas-induced apoptosis pathways. The 1295 edges represent

interactions, catalytic reactions, transcriptional regulation (for a total of 766 direct interactions) and known indirect causal regulation. No double causal inferences (relationships among three nodes) were available for this network.
 Performing BTR with NET-SYNTHESIS reduced the total

edge-number to 873. To focus on pathways that involve the 33 known T-LGL deregulated proteins, we designated vertices that correspond to proteins with no evidence of being changed during T-LGL as pseudo-vertices and deleted the label 'Y' for those edges whose both endpoints were pseudo-vertices. Recursively performing 'Reduction (faster)' BTR and 'Collapse degree-2 pseudonodes' of NET-SYNTHESIS until no edge/node could be further removed simplified the network to 267 nodes and 751 edges. Performing comprehensive PVC led to a drastic reduction to 38 vertices and 108 edges. The drawback of this dramatic simplification is that pairs of incoherent edges (two edges with opposite signs) can appear among pairs of nodes. While incoherent paths between pairs of nodes are often seen in biological regulatory networks, interpretation of incoherent edges is difficult without knowledge of the mediators of the two opposite regulatory mechanisms. The number of incoherent edge pairs ranged between 3 (when collapsing degree two pseudo-vertices only) and 19 (for comprehensive PVC). Thus, optimal simplification may require several alternative applications of the various options of PVC algorithms.

regulatory relationships between nodes, including protein

# 3.2 Synthesizing a network for abscisic acid(ABA)-induced stomatal closure

We have performed a comparison of the manually curated network for ABA-induced closure published in Li et al. (2006) with the output of NET-SYNTHESIS as reported in Albert et al. (2007). The input to NET-SYNTHESIS is a list of 140 interactions and causal inferences in ABA-induced closure published in Table S1 and Text S1 in Li et al. (2006). The complete list of causal relationships is given in Table 1 in the Supplementary Material. A detailed comparison of the two networks is available in Albert et al. (2007), here we briefly summarize the overall comparison of the two networks. The network of Li et al. (2006) has 54 vertices and 92 edges; our network has 57 vertices (3 extra pseudo-vertices) but 84 edges. The two networks have 71 common edges and identical strongly connected components. All the paths present in the (Li et al., 2006) reconstruction are present in our network as well. Thus, the two networks are highly similar and their divergence on a few edges is due not to algorithmic deficiencies but to human decisions. Finally, the entire network synthesis process was done within a few seconds by our software. A picture of our network is available as Figure 1 in the Supplementary Material.

### 4 CONCLUSION

The applications of NET-SYNTHESIS enable us to conclude that it can serve as a very important first step in formalizing the logical substrate of an inferred signal transduction network. We foresee its optimal application in conjunction with human expertise, as part of an interactive and iterative process. The NET-SYNTHESIS users would give the experimentally known information as input, then use the output network to augment the input information with additional facts or hypotheses, allowing them to simultaneously synthesize their knowledge and formalize their hypotheses regarding a signal transduction network.

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Conflict of Interest: none declared.

#### REFERENCES

- Albert, R. et al. (2007) A novel method for signal transduction network inference from indirect experimental evidence. J. Comp. Biol., 14, 927–949.
- Epling-Burnette, P.K. *et al.* (2004) ERK couples chronic survival of NK cells to constitutively activated Ras in lymphoproliferative disease of granular lymphocytes. *Oncogene*, 23, 9220–9229.
- Li,S. et al. (2006) Predicting essential components of signal transduction networks: a dynamic model of guard cell abscisic acid signaling. PLoS Biol., 4, e312, doi:10.1371/journal.pbio.0040312.
- Loughran, T.P., Jr (1993) Clonal diseases of large granular lymphocytes. *Blood*, **82**, 1–14.

interaction	critical	enzymatic	interaction	critical	enzymatic
$ABA \rightarrow SphK$	No	No	$ABA \rightarrow OST1$	No	No
$ABA \rightarrow CaIM$	No	No	$ABA \rightarrow InsP6$	No	No
$ABA \rightarrow Ca2+c$	No	No	$ABA \rightarrow NO$	No	No
$ABA \rightarrow InsP3$	No	No	$ABA \rightarrow AnionEM$	No	No
$ABA \dashv PEPC$	No	No	$ABA \dashv Malate$	No	No
$ABA \dashv HATPase$	No	No	$ABA \dashv RAC1$	No	No
$ABA \rightarrow PLD$	No	No	$ABA \rightarrow ROS$	No	No
$Ca2+c \dashv CaIM$	No	No	$Ca2+c \rightarrow KEV$	No	No
$Ca2+c \rightarrow AnionEM$	No	No	$InsP6 \rightarrow Ca2+c$	No	No
$InsP6 \rightarrow CIS$	No	No	$ROS \rightarrow CaIM$	No	No
$ROS \rightarrow Closure$	No	No	ROS ⊣ ABI1	No	No
$ROS \dashv KOUT$	No	No	$pHc \rightarrow KOUT$	No	No
$pHc \rightarrow ABI1$	No	No	$pHc \rightarrow ROS$	No	No
$pHc \rightarrow HATPase$	No	No	PA ⊣ ABI1	No	No
$PA \rightarrow Closure$	No	No	$PA \rightarrow ROS$	No	No
$NO \rightarrow Closure$	No	No	$NO \rightarrow AnionEM$	No	No
$NO \dashv KOUT$	No	No	$RAC1 \dashv Actin$	No	No
$RAC1 \dashv Closure$	No	No	$ABH1 \dashv AnionEM$	No	No
AnionEM $\dashv$ Malate	No	No	$ERA1 \rightarrow ROP10$	No	No
Depolarization $\dashv$ Ca2+c	No	No	$\text{GPA1} \rightarrow \text{PLD}$	Yes	No
$\text{Sph} \rightarrow \text{S1P}$	Yes	No	$InsPK \rightarrow InsP6$	Yes	Yes
$PLC \rightarrow DAG$	Yes	Yes	$PIP2 \rightarrow DAG$	Yes	No
$PLC \rightarrow InsP3$	Yes	Yes	$PIP2 \rightarrow InsP3$	Yes	No
$\mathrm{GC}  ightarrow \mathrm{cGMP}$	Yes	Yes	$GTP \rightarrow cGMP$	Yes	No
$ADPRc \rightarrow cADPR$	Yes	Yes	$NAD \rightarrow cADPR$	Yes	No
$NADPH \rightarrow NO$	Yes	No	Nitrite $\rightarrow$ NO	Yes	No
$\operatorname{Arg} \to \operatorname{NO}$	Yes	No	$NOS \rightarrow NO$	Yes	Yes
$NIA12 \rightarrow NO$	Yes	Yes	$NADPH \rightarrow ROS$	Yes	No
$\operatorname{Atrboh} \to \operatorname{ROS}$	Yes	Yes	$Ca2+ATPase \dashv Ca2+c$	Yes	No
$Ca2+c \rightarrow Ca2+ATPase$	Yes	No	HATPase ⊢ Depolarization	Yes	No
KOUT ⊢ Depolarization	Yes	No	KAP ⊢ Depolarization	Yes	No
AnionEM $\rightarrow$ Depolarization	Yes	No	$Ca2+c \rightarrow Depolarization$	Yes	No
$\text{KEV} \rightarrow \text{Depolarization}$	Yes	No	$\begin{array}{c} \text{RCN1} \rightarrow \text{NIA12} \\ \text{CLN1} \rightarrow \text{CLA12} \end{array}$	No	No
$CIS \rightarrow Ca2+c$	Yes	No	$CaIM \rightarrow Ca2+c$	Yes	No
Malate $\dashv$ Closure	Yes	No	GCR1 ⊣ GPA1	Yes	No
$ABA \rightarrow RCN1$	No	No	AnionEM $\rightarrow$ Closure	Yes	No
$KAP \rightarrow Closure$	Yes No	No	$KOUT \rightarrow Closure$	Yes No	No
$\begin{array}{l} \text{ERA1} \dashv \text{CaIM} \\ \text{cGMP} \rightarrow \text{CIS} \end{array}$	No	No No	$\begin{array}{c} \text{ABH1} \dashv \text{CaIM} \\ \text{cADPR} \rightarrow \text{CIS} \end{array}$	No	No No
$InsP3 \rightarrow CIS$	No	No	$\begin{array}{c} \text{CADIAC} \rightarrow \text{CIS} \\ \text{Ca2+c} \rightarrow \text{NOS} \end{array}$	No	No
$ROS \rightarrow (ABA \rightarrow Closure)$	-	-	$AnionEM \rightarrow (ABA \rightarrow Closure)$	-	NO
$PLC \rightarrow (ABA \rightarrow Closure)$	-	-	$SphK \rightarrow (ABA \rightarrow Closure)$	-	-
$SphK \rightarrow (ABA \rightarrow AnionEM)$	_	_	$\text{SphK} \rightarrow (\text{ABA} \rightarrow \text{S1P})$	_	_
$S1P \rightarrow (ABA \rightarrow Closure)$	_	-	$GPA1 \rightarrow (S1P \rightarrow AnionEM)$	_	_
$GPA1 \rightarrow (ABA \rightarrow ROS)$	-	_	$GCR1 \dashv (ABA \rightarrow Closure)$	_	_
$PLC \rightarrow (ABA \rightarrow Ca2+c)$	-	-	$cADPR \rightarrow (ABA \rightarrow Ca2+c)$	-	-
$NOS \rightarrow (ABA \rightarrow Closure)$	-	-	$NO \rightarrow (ABA \rightarrow Closure)$	-	_
$NO \rightarrow (ABA \rightarrow Closure)$	-	-	$NO \rightarrow (ABA \rightarrow AnionEM)$	-	_
$Ca2+c \rightarrow (NO \rightarrow AnionEM)$	-	-	$NO \rightarrow (Ca2+c \rightarrow CIS)$	-	_
$ADPRc \rightarrow (NO \rightarrow Ca2+c)$	-	-	$GC \rightarrow (NO \rightarrow Ca2+c)$	-	-
$KOUT \rightarrow (ABA \rightarrow Closure)$	-	-	$\text{GPA1} \rightarrow (\text{ABA} \rightarrow \text{AnionEM})$	-	-
$pHc \rightarrow (ABA \rightarrow Closure)$	-	-	$\text{ERA1} \dashv (\text{ABA} \rightarrow \text{AnionEM})$	-	-
$ERA1 \dashv (ABA \rightarrow Closure)$	-	-	ERA1 $\dashv$ (Depolarization $\rightarrow$ KOUT)	-	-
Atrboh $\rightarrow$ (ABA $\rightarrow$ Closure)	-	-	$Atrboh \rightarrow (ABA \rightarrow ROS)$	-	-
Atrboh $\rightarrow$ (ABA $\rightarrow$ Ca2+c)	-	-	$Atrboh \rightarrow (ABA \rightarrow CaIM)$	-	-
$ROS \rightarrow (ABA \rightarrow CaIM)$	-	-	$\text{NADPH} \rightarrow (\text{ABA} \rightarrow \text{CaIM})$	-	-
$\rm NAD \rightarrow (ABA \rightarrow CaIM)$	-	-	$ERA1 \dashv (ABA \rightarrow CaIM)$	-	-
$\text{ERA1} \dashv (\text{ABA} \rightarrow \text{Closure})$	-	-	$\text{RCN1} \rightarrow (\text{ABA} \rightarrow \text{Closure})$	-	-
$\text{RCN1} \rightarrow (\text{ABA} \rightarrow \text{AnionEM})$	-	-	$RCN1 \rightarrow (ABA \rightarrow Ca2+c)$	-	-
$OST1 \rightarrow (ABA \rightarrow Closure)$	-	-	$OST1 \rightarrow (ABA \rightarrow ROS)$	-	-
$PLC \rightarrow (ABA \rightarrow Closure)$	-	-	$Ca2+c \rightarrow (ABA \rightarrow Closure)$	-	-
AnionEM $\rightarrow$ (ABA $\rightarrow$ Closure)	-	-	$PLD \rightarrow (PC \rightarrow PA)$	-	-
$PLD \rightarrow (ABA \rightarrow Closure)$	-	-	$PLC \rightarrow (ABA \rightarrow Closure)$	-	-
$ABA \rightarrow (PLD \rightarrow PA)$	-	-	$ABA \rightarrow (PLD \rightarrow PA)$	-	-

Table 1. Regulatory interactions between ABA signal transduction pathway components (Li et al., 2006).

Table 1 (continued)									
interaction	critical	enzymatic	interaction	critical	$\mathbf{enzymatic}$				
$ROP2 \rightarrow (PA \rightarrow ROS)$	-	-	$Actin \rightarrow (ABA \rightarrow Closure)$	-	-				
$Ca2+c \rightarrow (ABA \rightarrow Actin)$	-	-	$RAC1 \dashv (ABA \rightarrow Closure)$	-	-				
$ROP10 \dashv (ABA \rightarrow Closure)$	-	-	$ROS \rightarrow (ABA \rightarrow Closure)$	-	-				
$\text{GCR1} \dashv (\text{ABA} \rightarrow \text{Closure})$	-	-	$GCR1 \dashv (S1P \rightarrow Closure)$	-	-				
$cADPR \rightarrow (Ca2+c \rightarrow CIS)$	-	-	AnionEM $\rightarrow$ (ABA $\rightarrow$ Closure)	-	-				
$CaIM \rightarrow (ABA \rightarrow KOUT)$	-	-	$cADPR \rightarrow (ABA \rightarrow KOUT)$	-	-				
$PLC \rightarrow (ABA \rightarrow KOUT)$	-	-	$ROS \rightarrow (ABA \rightarrow CaIM)$	-	-				
$Ca2+c \dashv (Depolarization \rightarrow KAP)$	-	-	pHc $\dashv$ (Depolarization $\rightarrow$ KAP)	-	-				
$ABH1 \dashv (ABA \rightarrow Closure)$	-	-	$ABH1 \dashv (ABA \rightarrow Ca2+c)$	-	-				
$ROS \rightarrow (ABA \dashv HATPase)$	-	-	$ABI1 \dashv (ABA \rightarrow AnionEM)$	-	-				
$ABI1 \dashv (ABA \rightarrow ROS)$	-	-	$ABI1 \dashv (ABA \rightarrow Ca2+c)$	-	-				
AtPP2C $\dashv$ (ABA $\rightarrow$ Closure)	-	-	$Ca2+c \rightarrow (PLC \rightarrow InsP3)$	-	-				
$\text{GPA1} \rightarrow \text{AGB1}$	No	No	$AGB1 \rightarrow GPA1$	No	No				
AtPP2C $\dashv$ Closure	No	No	$NO \rightarrow ADPRc$	No	No				
$Ca2+c \rightarrow HATPase$	No	No	$ABI1 \dashv Atrboh$	No	No				
$\rm NO \rightarrow GC$	No	No	$ABA \rightarrow pHc$	No	No				
$PA \rightarrow ROP2$	No	No	$PEPC \rightarrow Malate$	Yes	Yes				
$ABI1 \dashv (ABA \rightarrow ROS)$	-	-	$ABA \rightarrow PLC$	No	No				
Depolarization $\rightarrow \text{KOUT}$	Yes	No	Depolarization $\rightarrow$ KAP	Yes	No				
Depolarization $\dashv$ CaIM	Yes	No	$ABI1 \rightarrow (ABA \dashv RAC1)$	-	-				
$InsPK \rightarrow (ABA \rightarrow AnionEM)$	-	-	$InsPK \rightarrow (ABA \rightarrow InsP6)$	-	-				
$S1P \rightarrow GPA1$	No	No	l · · · · · · · · · · · · · · · · · · ·						

Table 1. Regulatory interactions between ABA signal transduction pathway components (Li et al., 2006).

## Supplementary Information: Description of the Network Synthesis Procedure

Here we sketch the framework of the network synthesis procedure employed in NET-SYNTHESIS. A complete description can be found in [Albert *et al.*, 2007].

The procedure applies to directed graphs G = (V, E) with an edge labeling function  $w : E \mapsto \{0, 1\}$ . Biologically, edge labels 0 and 1 in edges  $u \xrightarrow{0} v$  and  $u \xrightarrow{1} v$  correspond to "u promotes v" and "u inhibits v", respectively.

The parity (sign) of a path P from vertex u to vertex v is  $\sum_{e \in P} w(e) \pmod{2}$ . A path of parity 0 is called a path of *even* parity, or positive sign. A path of parity 1 is called a path of *odd* parity, or negative sign. The notation  $u \stackrel{x}{\Rightarrow} v$  denotes a path from u to v of parity  $x \in \{0, 1\}$ .

For a subset of edges  $E' \subseteq E$ , reachable(E') is the set of all ordered triples (u, v, x) such that  $u \stackrel{x}{\Rightarrow} v$  is a path of the subgraph (V, E'). The binary transitive reduction (BTR) problem is defined as follows:

**Instance:** A directed graph G = (V, E) with an edge labeling function  $w : E \mapsto \{0, 1\}$  and a set of critical edges  $E_{\text{critical}} \subseteq E$ .

Valid Solutions: A subgraph G' = (V, E') where  $E_{\text{critical}} \subseteq E' \subseteq E$  and reachable(E') = reachable(E). Objective: Minimize |E'|.

Intuitively, the BTR problem is useful for determining the sparsest graph consistent with a set of experimental observations. The set of "critical edges" represent edges which are known to be direct interactions with concrete evidence.

The pseudo-vertex collapse (PVC) problem is defined as follows:

**Instance:** A directed graph G = (V, E) with an edge labeling function  $w : E \mapsto \{0, 1\}$  and a subset  $V' \subset V$  of vertices called pseudo-vertices. The vertices in  $V \setminus V'$  are called "real" vertices.

### **Definition:**

- For any vertex v, let  $in(v) = \{(u, x) \mid u \stackrel{x}{\Rightarrow} v, x \in \{0, 1\}\} \setminus \{v\}$  and let  $out(v) = \{(u, x) \mid v \stackrel{x}{\Rightarrow} u, x \in \{0, 1\}\} \setminus \{v\}$ .
- Collapsing two vertices u and v is permissible provided both are not "real" vertices and in(u) = in(v) and out(u) = out(v).
- If permissible, the collapse of two vertices u and v creates a new vertex w, makes every incoming (resp. outgoing) edges to (resp. from) either u or v an incoming (resp. outgoing) edge from w, removes any parallel edge that may result from the collapse operation and also removes both vertices u and v.

Valid Solutions: A graph G'' = (V'', E'') obtained from G by a sequence of permissible collapse operations.

**Objective:** Minimize |V''|.

Intuitively, the PVC problem is useful for reducing the pseudo-vertex set to the the minimal set that maintains the graph consistent with all indirect experimental observations. As in the case of the BTR problem, our goal is to minimize false positive (spurious) inferences of additional components in the network.

The main network synthesis steps employed in NET-SYNTHESIS are the following:

- 1. Incorporate single causal inferences and biochemical interactions as labeled edges, noting the critical edges corresponding to direct interactions.
- 2. Perform a binary transitive reduction to eliminate spurious inferred edges (*i.e.*, edges that can be explained by paths of the same label).
- 3. Incorporate double causal relationships  $A \xrightarrow{x} (B \xrightarrow{y} C)$  by (i) adding a new edge  $A \xrightarrow{x} B$  if  $B \xrightarrow{y} C$  is an existing critical edge, (ii) doing nothing if existing paths in the network already explain the relationship, or (iii) adding a new pseudo-vertex P and three new edges  $A \xrightarrow{x} P$ ,  $B \xrightarrow{a} P$  and  $P \xrightarrow{b} C$ . To correctly incorporate the parity of the  $A \xrightarrow{x+y} (\text{mod } 2) C$  relationship, positive  $B \xrightarrow{y} C$  paths will be broken into two positive edges, while negative paths will be broken into a positive edge (a = 0) and a negative edge (b = 1), summarized in a concise way by the equation  $b = a + b = y \pmod{2}$ .
- 4. Perform pseudo-vertex collapse to reduce unnecessary redundancy in the resulting graph.
- 5. Perform a second round of binary transitive reduction to eliminate any redundant edges created by pseudo-vertex collapse.

Intuitively speaking, the approach is to first expand the network by the addition of the pseudo-vertices at the intersection of the two paths corresponding to double (three-node) causal inferences, then to use the additional information available in the network to collapse these pseudo-vertices, *i.e.*, to identify them with real vertices or with each other.

NET-SYNTHESIS offers two additional procedures of interest in certain biological networks. If chosen by the user, indirect regulation of a product of an enzymatic reaction will be interpreted as regulation of the enzyme, implemented as the action "Enzymatic edges". The user can also designate any node as a pseudonode by prepending a \* to the name of the node, and then perform PVC either on the whole network or on degree-two pseudo-nodes only. An example of a set of input interactions for a network synthesis approach is given in the file http://www.cs.uic.edu/~dasgupta/network-synthesis/sample-input-file.txt on the NET-SYNTHESIS webpage. This file also provides a suggested sequence of actions for this network.

### References

[Albert et al., 2007] R. Albert, B. DasGupta, R. Dondi, et al. (2007). A Novel Method for Signal Transduction Network Inference from Indirect Experimental Evidence, Journal of Computational Biology, 14 (7), 927-949.

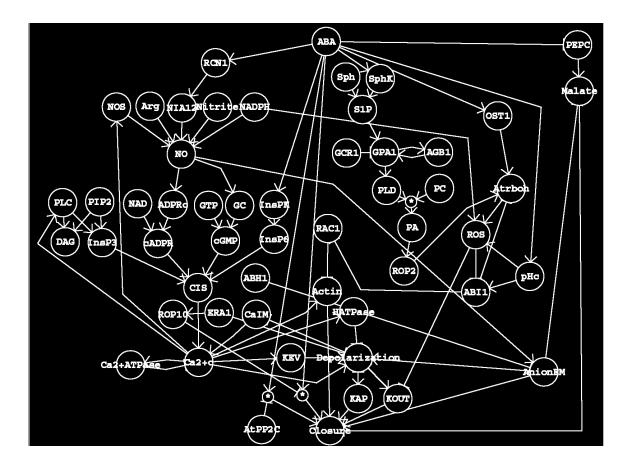


Figure 1: The guard cell signal transduction network for ABA-induced stomatal closure produced by NET-SYNTHESIS.